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**Characteristics of a Three Phase  
and Single Phase Induction Motor**

**Electrical Engineering**

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**CHARACTERISTICS OF A  
THREE PHASE AND SINGLE PHASE INDUCTION MOTOR**

**BY**

**JEFFERSON HALL BELT  
AND  
NORRIS FAY MURRAY**

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**T H E S I S**

**FOR THE**

**DEGREE OF BACHELOR OF SCIENCE**

**IN**

**ELECTRICAL ENGINEERING**

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**COLLEGE OF ENGINEERING**

**UNIVERSITY OF ILLINOIS**

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May 28, 19012

THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

JEFFERSON HALL BELT AND NORRIS FAY MURRAY

ENTITLED CHARACTERISTICS OF A

THREE PHASE AND SINGLE PHASE INDUCTION MOTOR

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF BACHELOR OF SCIENCE IN ELECTRICAL ENGINEERING

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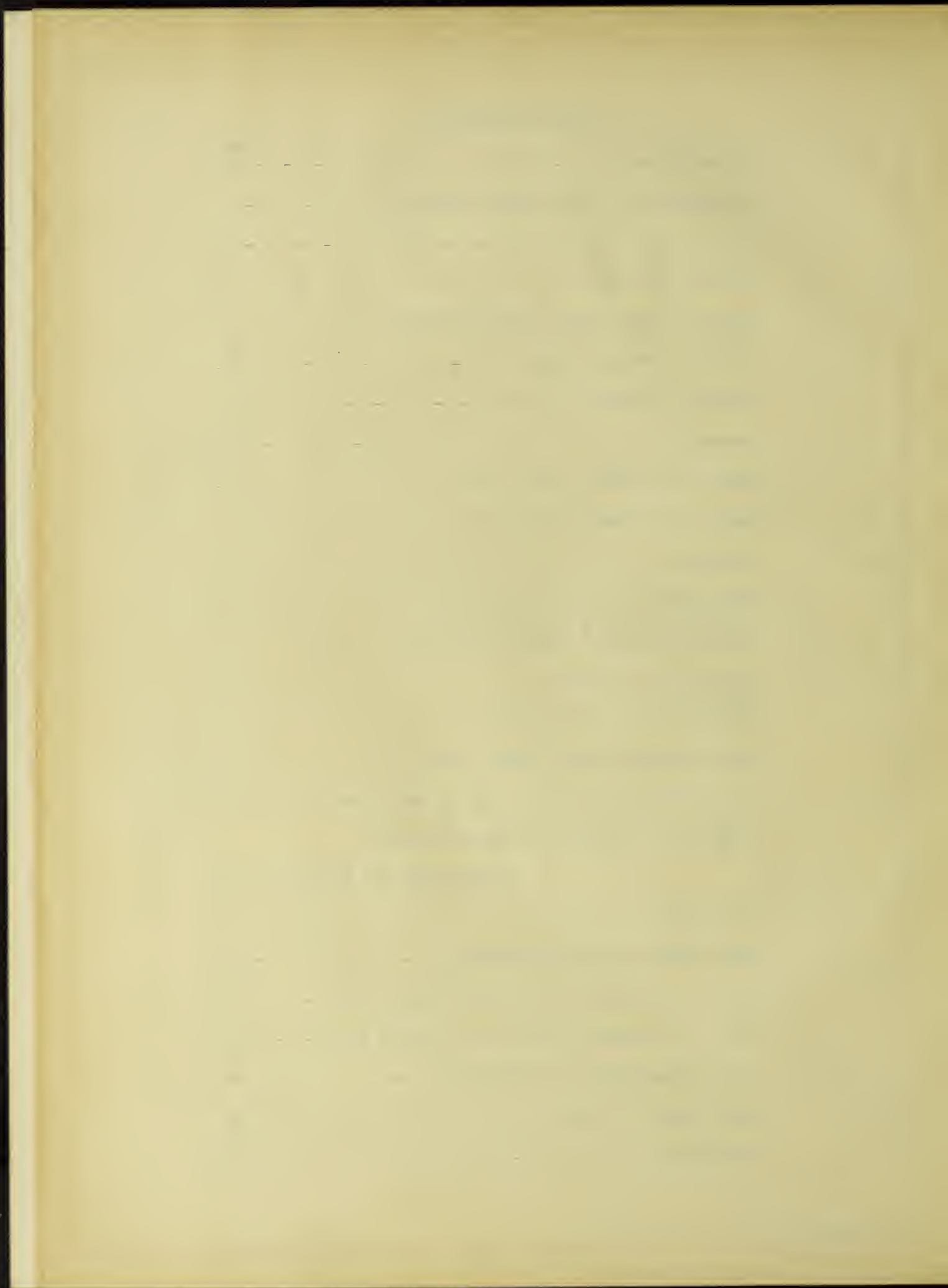
CHARACTERISTICS OF A  
THREE PHASE AND SINGLE  
PHASE INDUCTION MOTOR.





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CHARACTERISTICS OF A  
THREE PHASE AND SINGLE  
PHASE INDUCTION MOTOR.

INTRODUCTION.

It is interesting to know what may be expected of a three phase induction motor, if it is desired to use it as a single phase motor. It may happen that a three phase motor of a certain rated capacity, can be supplied only with single phase current; it is then of considerable importance to know what part of its rated capacity it will deliver under these conditions of operation. Again, a three phase induction motor running normally from three phase power, may have one of its phases open circuited or one of its supply lines disconnected. Since the torque of a single phase motor is less than that of a three phase motor, it is a question whether it will fall out of step, or work under overload until it burns out. In order to know what to expect under such circumstances, the characteristics of the induction motor, both when running normally three phase and under single phase conditions, must be determined.

The scheme followed in working out this problem was, first, to determine the theoretical characteristics of the motor, under the two condition of operation





by the analytical method as developed by Steinmetz in his *Elements of Electrical Engineering*, pp.356 et seq. The constants of the motor used in these calculations were determined by actual test of the motor under consideration. After the results were obtained in the above manner, prony brake tests were made to determine the actual operating characteristics of the motor. These were compared with the theoretical data, taking into consideration the inaccuracy of prony brake tests, and an effort was made to account for the differences which exist between the results of the two methods. The problem then is, to determine the operating characteristics of a three phase induction motor when running as a three phase motor and as a single phase motor.

The motor tested was a 110 volt, 6 pole, 60 cycle, 1200 R.P.M., 5 H.P., Y Y connected, General Electric Co. No. 90034, three phase induction motor.

#### DETERMINATION OF THREE PHASE CONSTANTS.

The principle constants of the induction motor used in calculating its characteristics are:-

Primary resistance	$= r_0$
Secondary resistance	$= r_1$
Primary self inductive reactance	$= x_0$
Secondary self inductive reactance	$= x_1$
Magnetizing current	$= I_m$
Hysteresis or core loss current	$= I_h$

For a three phase motor, the constants are determined by the "running





light" and " blocked rotor " tests.

### RUNNING LIGHT TEST.

The motor is allowed to run at no load, and is supplied with various voltages at normal frequency, ranging from 25 per cent above normal to a value at which the motor will just continue running. Beginning with the highest value of voltage, current and watt intake are measured for each successive value until the motor begins to stop. The losses are separated as follows;- referring to fig. 1,

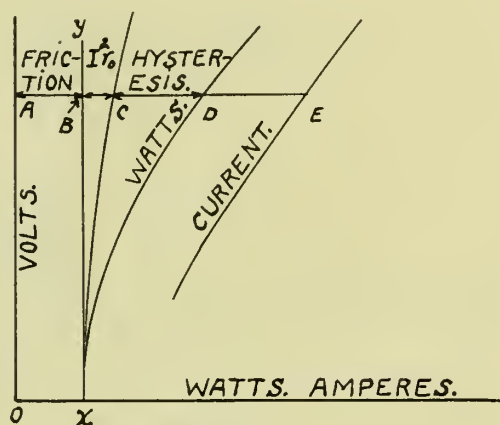
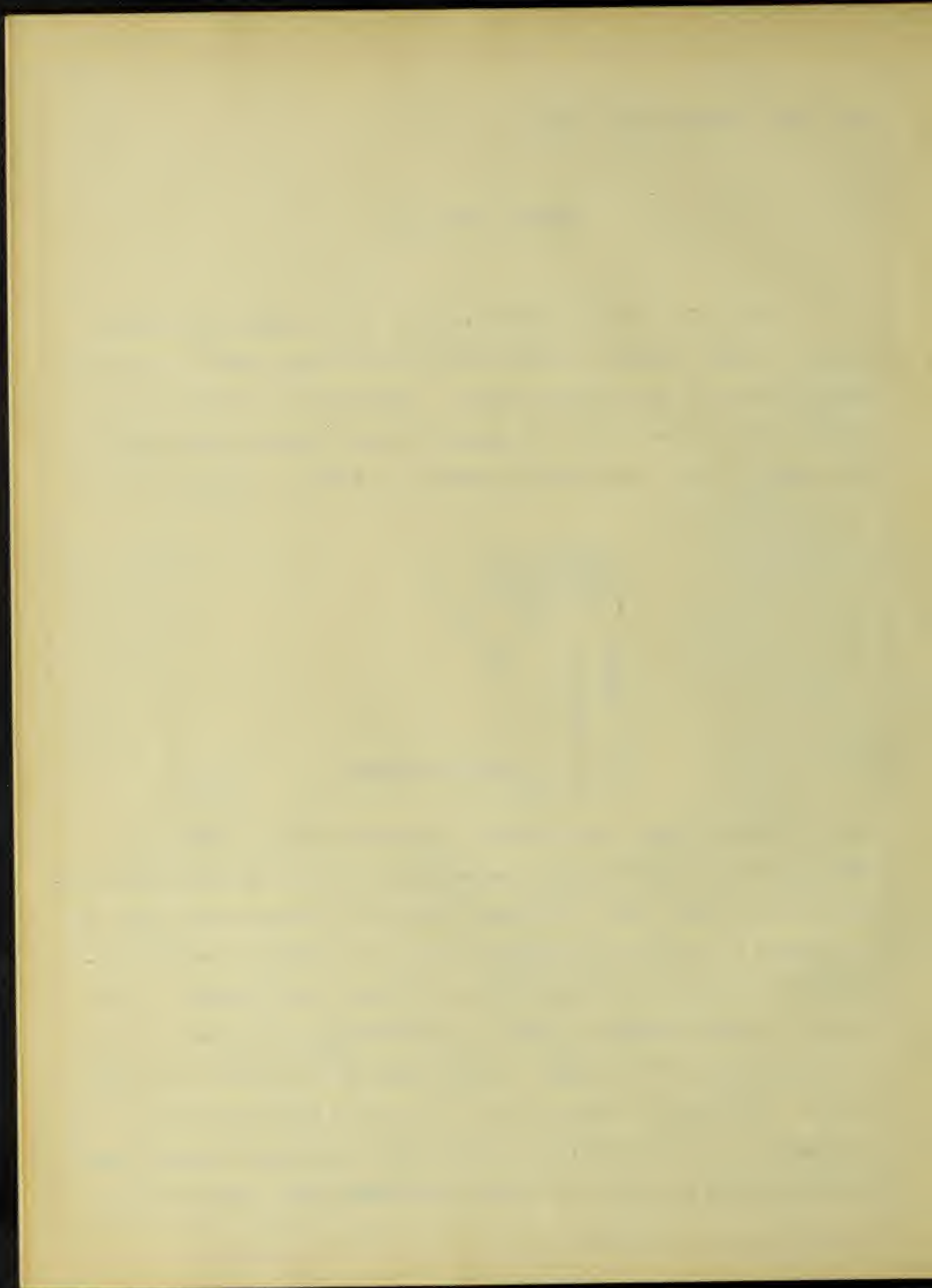


Fig. 1.

curves are plotted between volts and watts and between volts and amperes. The losses due to hysteresis at constant frequency vary with the flux density and therefore with the counter e.m.f. The copper losses with the motor running light may be calculated from primary resistance and current, the loss in the secondary being very small due to the low resistance and the small currents induced. As the voltage is reduced, the speed, current, and electrical losses decrease. If the motor could be run at zero voltage, the only losses would be those of mechanical friction. Draw the line XY and lay off to the right of this the  $I^2r$  curve as calculated from the primary currents and resistance. Horizontal distances between this curve and the watt curve will represent hysteresis losses. Draw the normal





voltage line AE intersecting the curves. The portions of this line between the volt axis and watt curve represents to scale the losses at normal voltage as explained above. The intersection with the ampere curve gives the normal exciting current. If  $W_h'$  represents the watts lost per phase due to hysteresis, and  $E'$  equals volts per phase, then

$$I_h = W_h' / E'$$

is the hysteresis current per phase.

$I_{00}$  = no load or exciting current per phase.

$I_m$  = wattless magnetizing current per phase.

$Y = g + jb$  = exciting admittance per phase.

$g$  = conductance per phase.

$b$  = susceptance per phase.

Then vectorially

$$I_{00} = I_h + jI_m$$

or numerically

$$I_{00} = \sqrt{I_h^2 + I_m^2}$$

$I_{00}$  is read from the curves and  $I_h$  has been determined, hence : -

$$I_m = \sqrt{I_{00}^2 - I_h^2}$$

$$g = I_h / E'$$

$$b = I_m / E'$$

#### BLOCKED ROTOR TEST.

Considering now the blocked rotor test, let the rotor be rigidly fixed in some convenient manner to prevent movement. Beginning with very low values, impress various voltages upon the motor and read corresponding amperes and watts until 50 per cent overload current flows. At the low voltages required, the hysteresis loss will be negligible, hence the watt input represents primary and

WILLIAM H. HARRIS

and secondary copper losses.

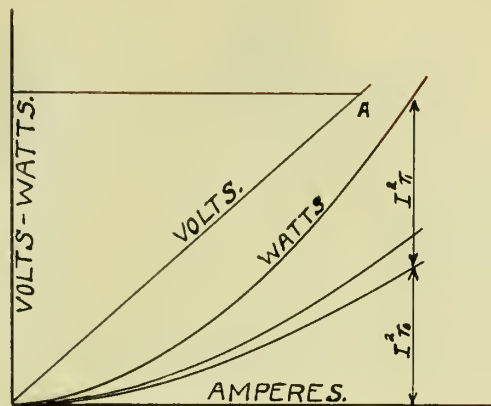


Fig.2.

Referring to fig.2 plot watts and volts against amperes. Calculate and plot the primary  $I^2 r$  curve. Vertical distances between this and the watt curve represent secondary  $I^2 r$  losses. These may then be plotted. The curve between volts and amperes is almost a straight line and if produced, intersects the normal voltage line at a point A, which represents the "starting current" or the current which would flow if normal voltage were impressed. Let

$$z_0 = r_0 - jx_0 \quad \text{primary impedance.}$$

$$z_1 = r_1 - jx_1 \quad \text{secondary impedance.}$$

$$Z = z_0 + z_1$$

Since total watts per phase,  $W$ , equal the  $I^2 r$  loss of primary and secondary per phase

$$W/I^2 = R \quad \text{per phase of primary and secondary.}$$

$$R - r_0 = r_1$$

$$E'/I' = Z = R - jX.$$

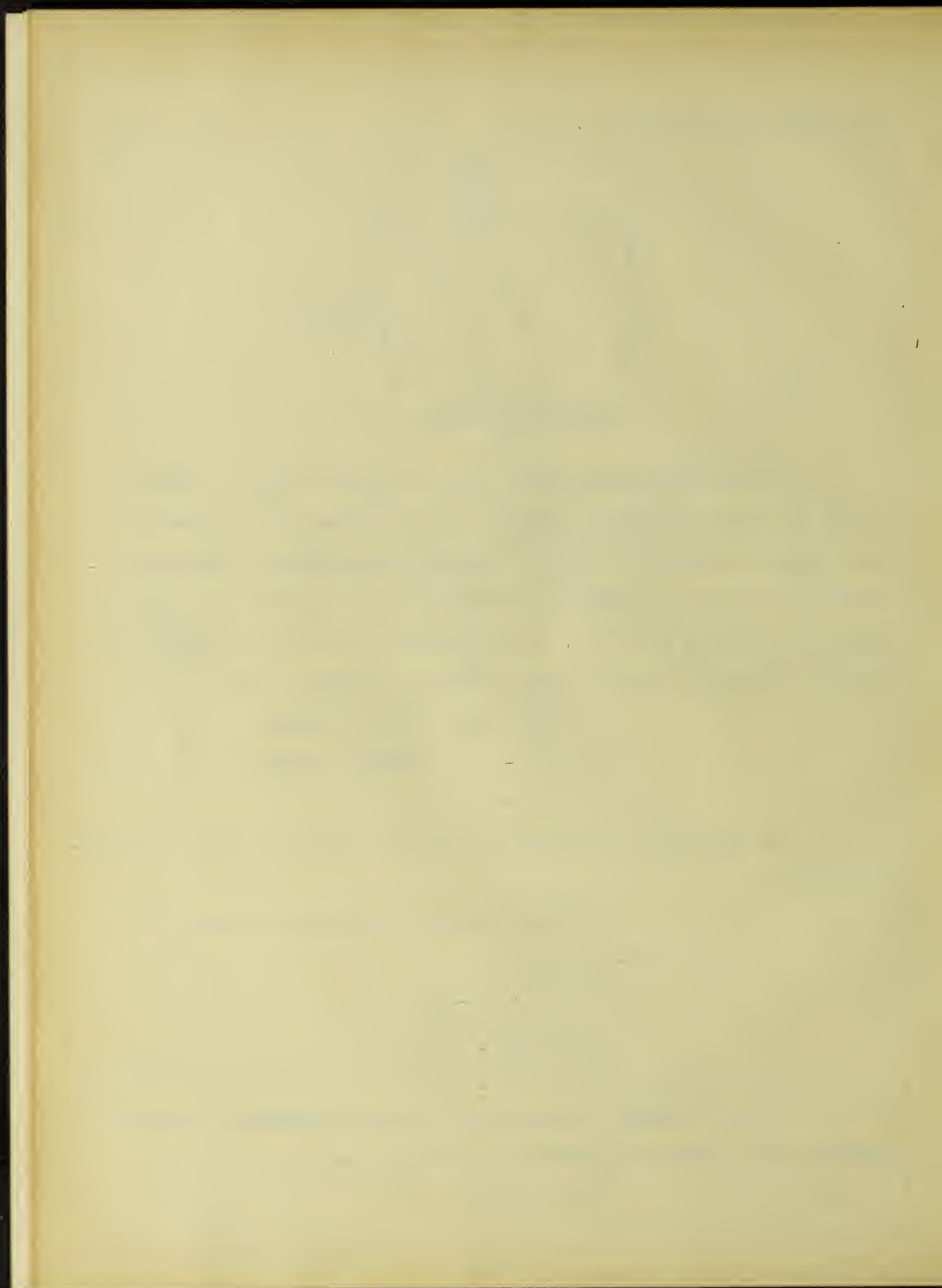
$$Z = \sqrt{R^2 + X^2}$$

$$X = \sqrt{Z^2 - R^2} \quad \text{the reactance per phase}$$

of both primary and secondary. Since there is no way to separately determine the reactances of the primary and secondary it is divided equally, thus,

$$x_0 = x_1 = X/2.$$





A mean value of R and Z should be taken from the blocked rotor readings and used to calculate X.

#### THEORY OF THREE PHASE INDUCTION MOTOR.

Having thus determined the constants of the three phase induction motor, the analytic equations for calculating its characteristics will now be developed, after the method of Steinmetz.

With the notation the same as above and representing values per phase;-

$E_0$  = impressed voltage.

$Y$  =  $g + jb$  = primary exciting admittance with open <sup>secondary</sup> circuit.

$e$  = counter generated e.m.f. of motor then,

$ge$  = power component of exciting current or core loss current.

$be$  = wattless component of exciting current or magnetizing current.

$z_0 = r_0 - jx_0$  = primary self inductive reactance.

$z_1 = r_1 - jx_1$  = secondary self inductive reactance, considering a one to one ratio of turns between primary and secondary windings.

$s$  = slip in per cent of synchronous speed as unity.

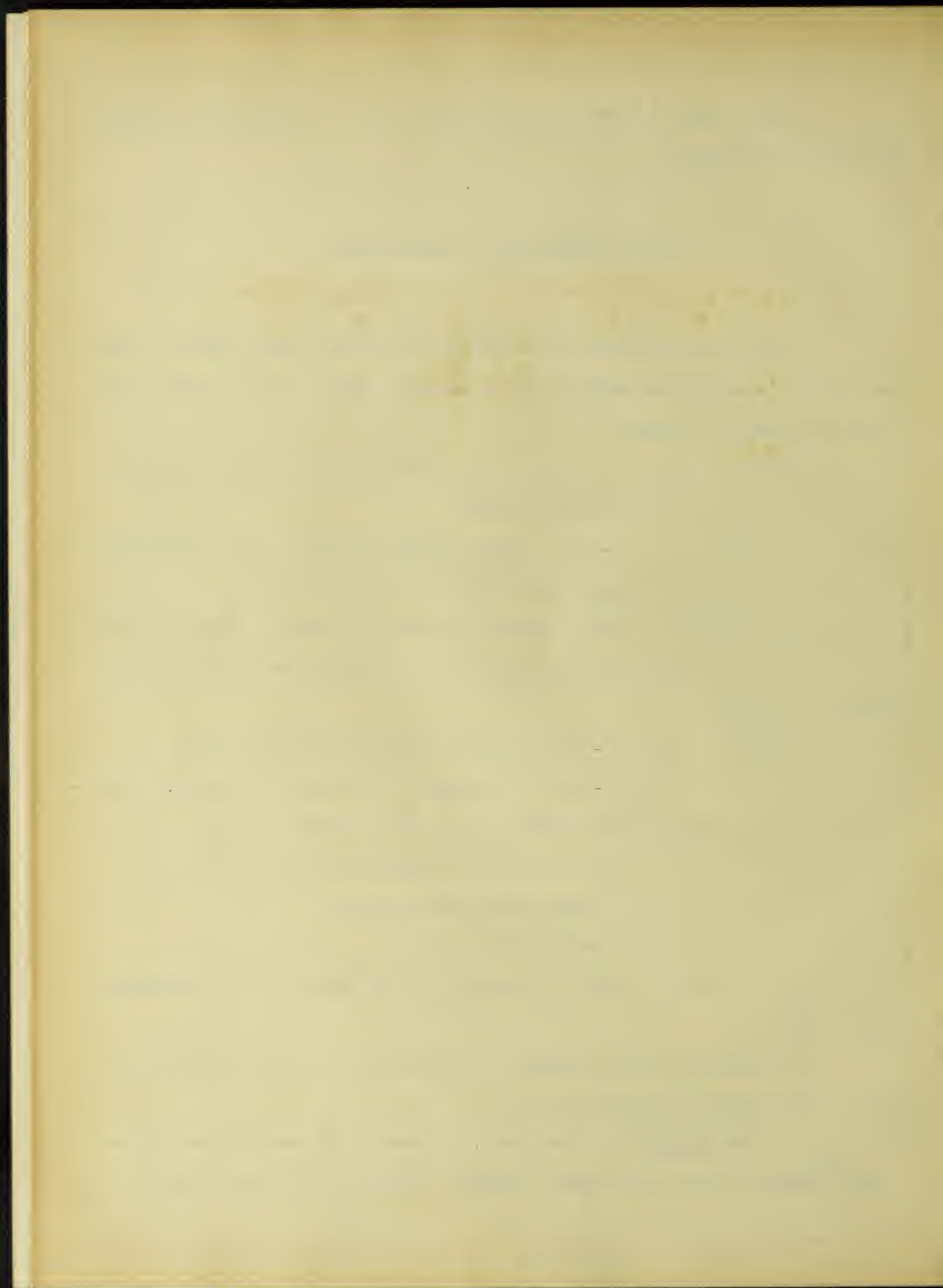
$s = 0$  denotes synchronous rotation.

$s = 1$  denotes stand still.

$1 - s$  = speed of secondary or rotor in per cent of synchronous speed.

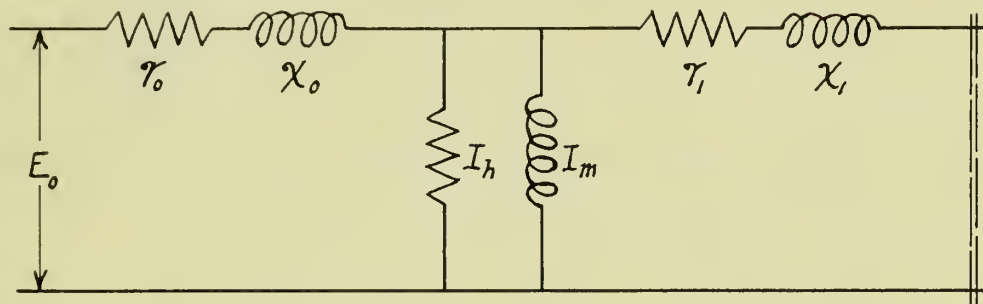
The induction motor resembles to a great extent, the transformer and may be represented by a diagram as in fig. 3.

Thus when  $s$  equals one the speed, one minus  $s$ , is zero and there is no motion between primary and secondary members. The condition is then that of a





transformer at short circuit and the secondary is subjected to the full frequency of the revolving polyphase field. Currents are induced in the secondary which oppose the motion of the primary field and thus a torque is exerted between the primary and secondary. If the secondary is allowed to move, it tends to travel



at the same speed as the primary field and the relative motion between the primary field and secondary conductors, and thus the cutting of the primary field by the secondary conductors, becomes less and less until at synchronous speed of secondary it would be zero. That is, the frequency of the secondary currents varies directly with the slip  $s$ , then,

$sf$  = frequency of secondary currents.

where  $f$  = frequency impressed upon the primary; therefore

$se$  = e.m.f. generated in secondary.

Since the reactance varies with the frequency, at any slip  $s$ , and frequency  $sf$ , the secondary reactance would be

$$x_1^s = sx_1,$$

and secondary impedance would be

$$z_1^s = r_1 - jsx_1.$$

hence the secondary current is

$$I_1 = \frac{se}{z_1^s} = \frac{se}{(r_1 - jsx_1)} = e \left( \frac{sr_1}{r_1^2 + s^2x_1^2} + \frac{js^2x_1^2}{r_1^2 + s^2x_1^2} \right) = e(a_1 + ja_2)$$

$$a_1 = \frac{sr_1}{r_1^2 + s^2x_1^2} \quad a_2 = \frac{s^2x_1^2}{r_1^2 + s^2x_1^2}$$



The primary exciting current is

$$I_{00} = I_h + jI_m = eY = e(g + jb)$$

and the total primary current is the sum of the secondary and exciting currents,

$$I_o = e[(a_1 + ja_2) + (g + jb)] = e(b_1 + jb_2)$$

$$b_1 = a_1 + g$$

$$b_2 = a_2 + b$$

The impressed e.m.f. ,  $E_o$  is consumed in the primary impedance  $z_o$ , and in overcoming the counter generated e.m.f. hence vectorially,

$$\begin{aligned} E_o &= e + I_o Z_o = e[1 + (b_1 - jb_2)(r_o - jx_o)] \\ &= e(c_1 + jc_2) \end{aligned}$$

$$c_1 = 1 + b_1 r_o + b_2 x_o$$

$$c_2 = b_2 r_o - b_1 x_o$$

Numerically

$$E_o = e\sqrt{c_1^2 + c_2^2}$$

whence the counter generated e.m.f. of the motor is

$$e = \frac{E_o}{\sqrt{c_1^2 + c_2^2}}$$

where  $E_o$  is the numerical value of the impressed e.m.f..

The value of  $e$  may now be substituted in the equation for primary current and numerically,

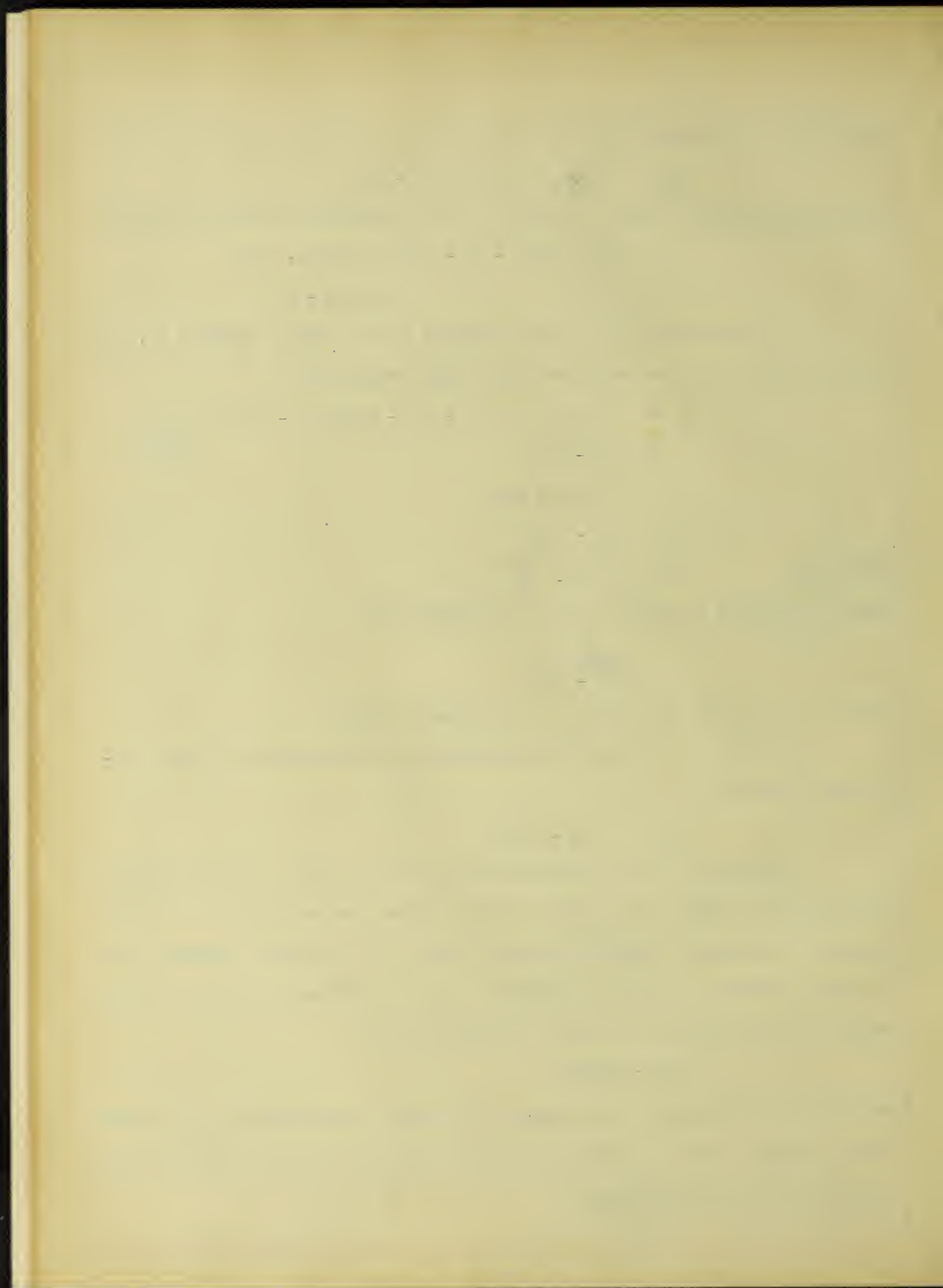
$$I_o = e\sqrt{b_1^2 + b_2^2}$$

The torque of the induction motor, or of any motor, is proportional to the product of the flux interlinked with both primary and secondary and with the component of secondary magnetomotive force, which is in time phase therewith, but in space quadrature. Since the generated e.m.f. is proportional to the mutual magnetic flux or rather to its rate of change, that is

$$e = -n \frac{d\phi}{dt}$$

and thus lags  $90^\circ$  in time, the torque of the motor is proportional to the product of the generated e.m.f. and the component of secondary current which is in quadrature with it in time and space.





$$I_1 = e(a_1 + ja_2)$$

is the secondary current corresponding to the generated e.m.f.,  $e$  and therefore the secondary current in quadrature with  $e$  is

$$jI_1 = e(ja_1 - a_2)$$

and the component of this current in time quadrature with  $e$  is  $a_1 e_1$ . The torque is then proportional to

$$e \cdot a_1 e \text{ or } T_s = e^2 a_1.$$

$T_s$  is the product of current and voltage and is thus a power. It is the power the motor would give if it ran at synchronous speed. Since the motor runs at a speed  $1 - s$ , less than synchronism, the power developed by the secondary is

$$P_1 = (1 - s)T_s.$$

This takes no account of friction and windage. Let  $F$  be this factor, then mechanical output in watts is  $P_1 - F$ .

The power intake of the primary,  $P_o$ , is the sum of the products of the inphase components of e.m.f. and current.

$$\text{Primary e.m.f.} = E_o = e(c_1 + jc_2)$$

$$\text{Primary current} = I_o = e(b_1 + jb_2)$$

$$P_o = ec_1 \cdot eb_1 + ec_2 \cdot eb_2 = e(b_1c_1 + b_2c_2)$$

The primary volt-amperes or apparent power,  $P_a$ , is

$$P_a = E_o I_o$$

$$\text{Eff.} = \frac{\text{net output}}{\text{input}} = \frac{P_1 - F}{P_o}.$$

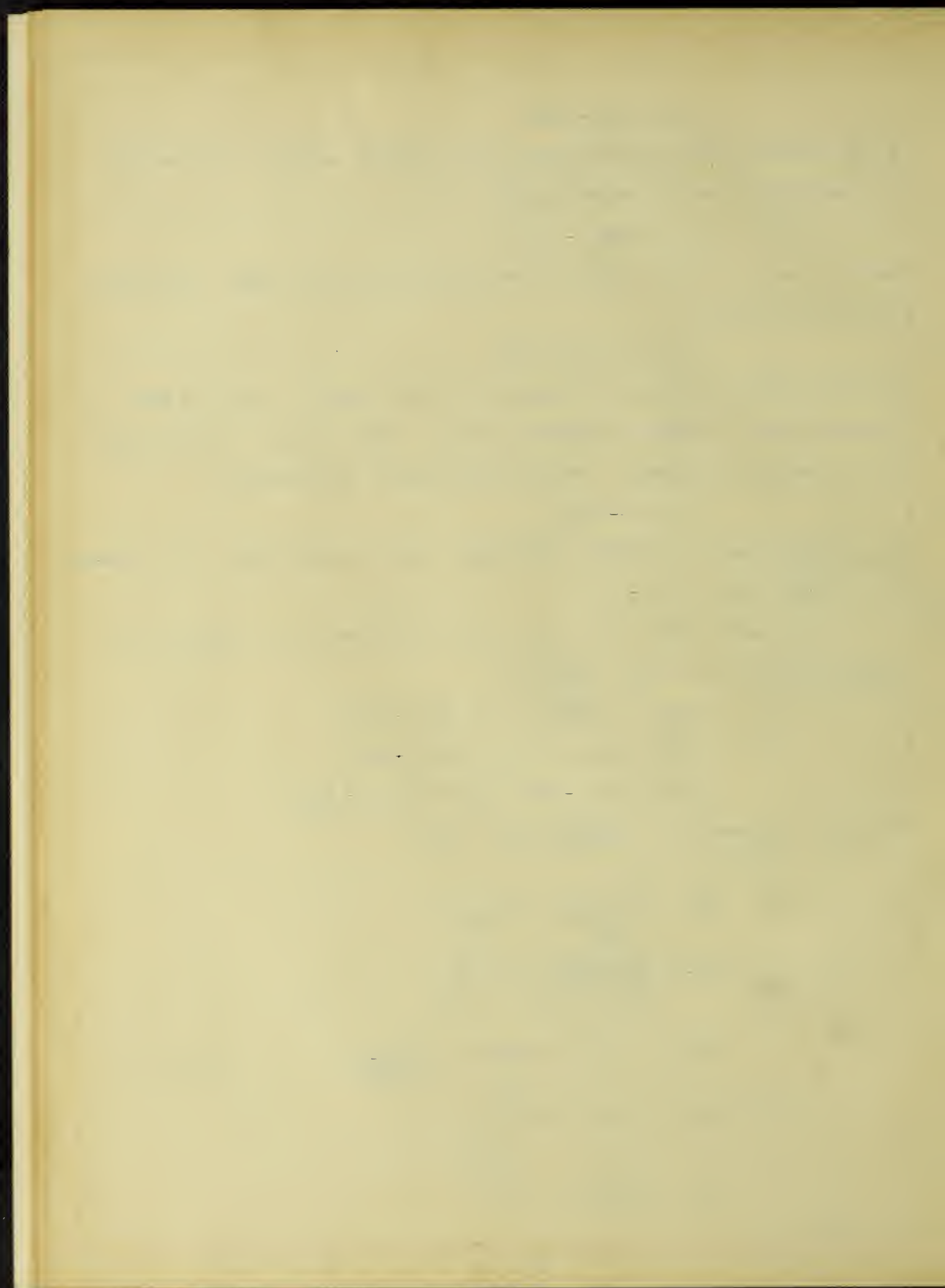
$$\text{P.F.} = \frac{\text{true power}}{\text{apparent watts}} = \frac{P_o}{P_a}$$

$$\text{H.P.} = 3 \cdot \text{H.P. per phase} = \frac{3(P_1 - F)}{746}.$$

To find torque in lb.ft. let

$$n = \text{R.P.S.}$$

$$T = \text{torque in lb. ft.}$$





Then the ft. lb. of work done per sec.

$$= 2 \pi n T$$

$$T_s = \frac{2 \pi n T \cdot 746.}{550}$$

$$T = \frac{550 T_s}{2 \pi n T \cdot 746.}$$

but  $f = \frac{\text{no. poles} \cdot n}{2} = pn/2$

hence  $n = 2f/p$

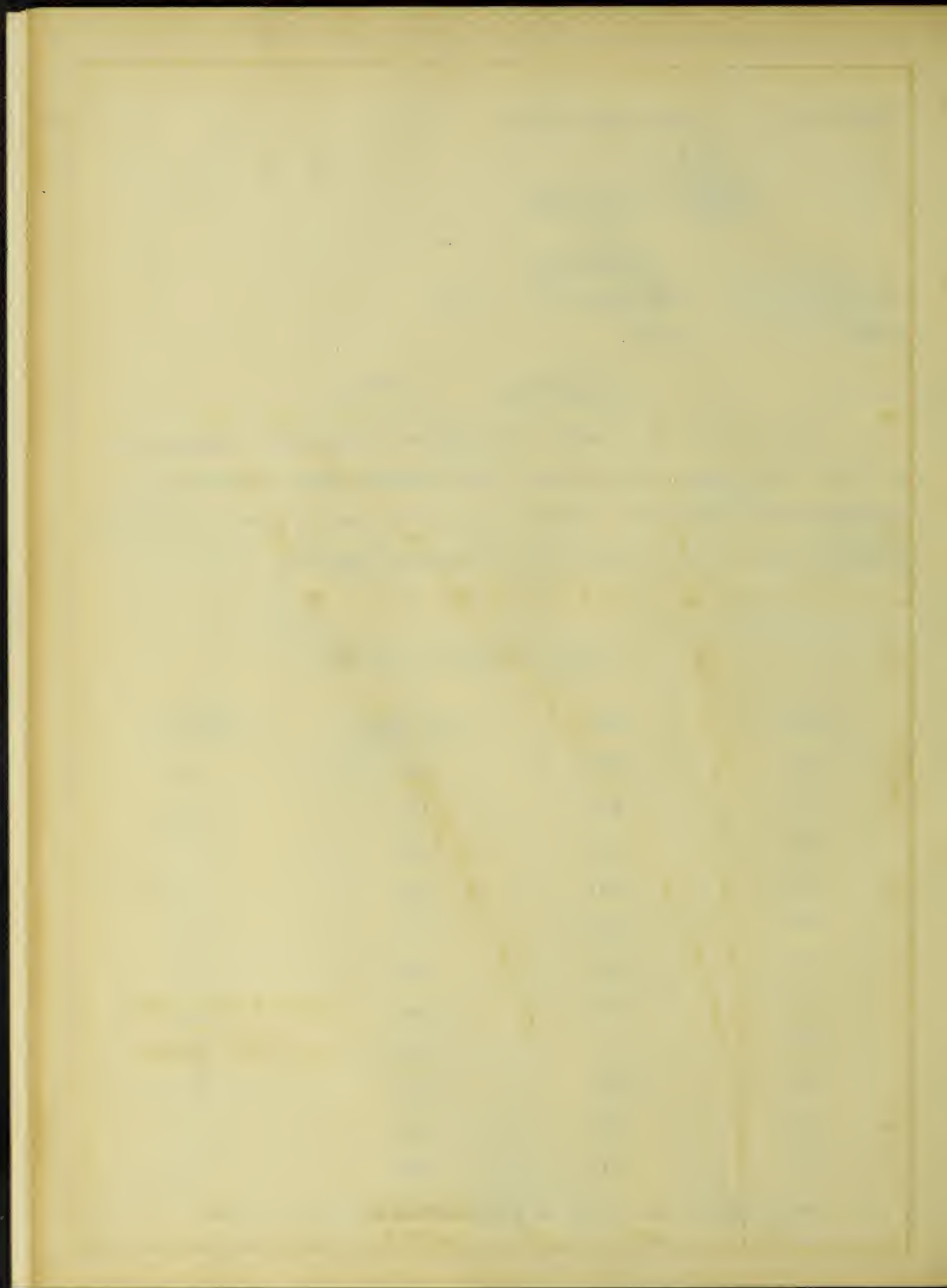
$$T = \frac{550 T_s p}{2 \pi \cdot 746 \cdot 2f} = .023 T_s$$

The constants of the motor having been determined by the running light and blocked rotor tests, they are used in the equations above to calculate the characteristics of the motor. The data taken in this test is given in the following pages together with the curves and the calculated constants.

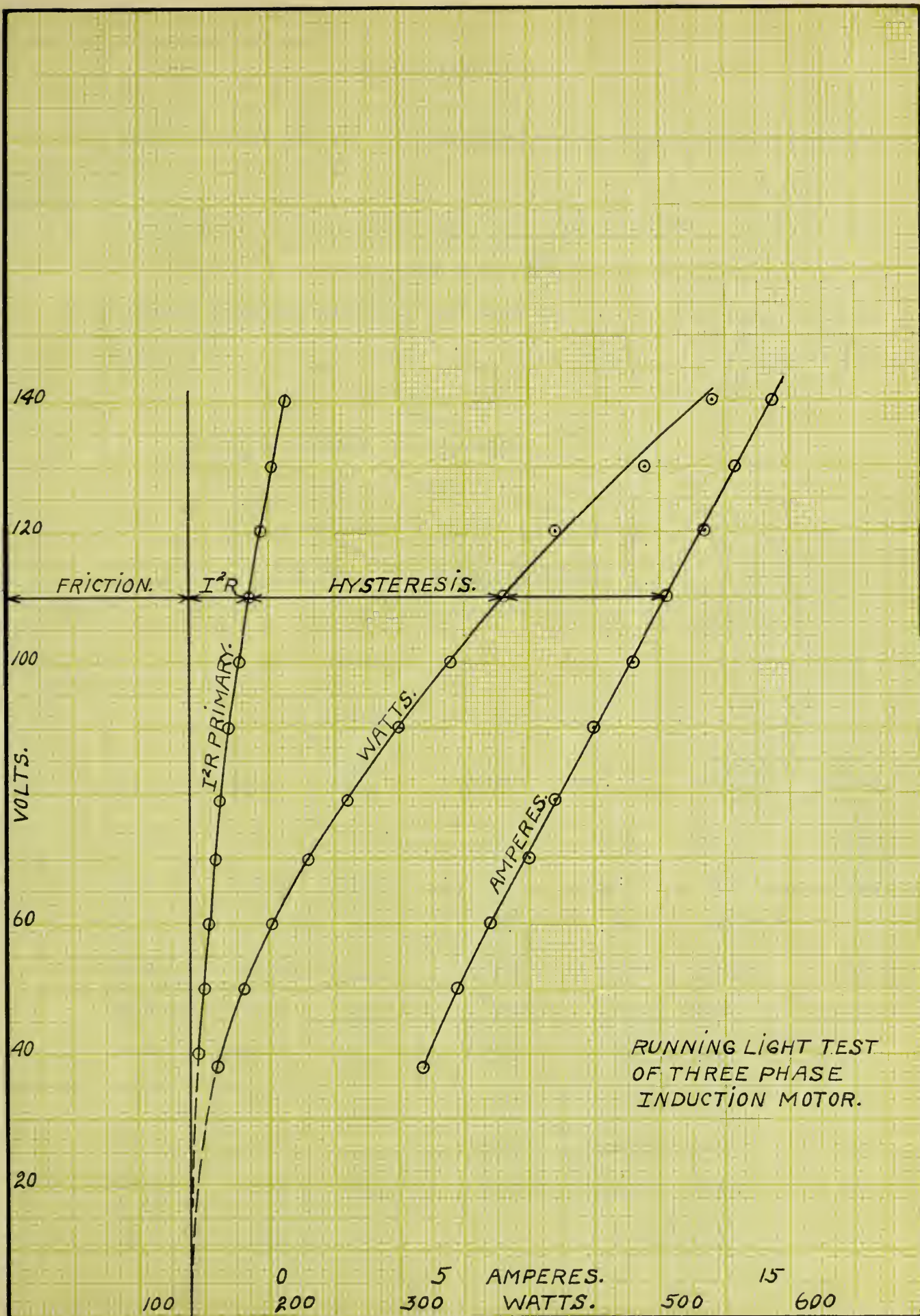
#### RESULTS OF RUNNING LIGHT TEST.

E volts	I amps.	W total watts.	I <sup>2</sup> r total pri.
140	15.5	540	72.
130	14.3	490	61.5
120	13.4	420	54.
110	12.2	380	44.7
100	11.2	340	37.8
90	10.	300	30.
79	8.8	260	23.3
70	8	230	19.2
60	6.8	200	13.8
50	5.8	180	10.1
38	4.7	160	6.6

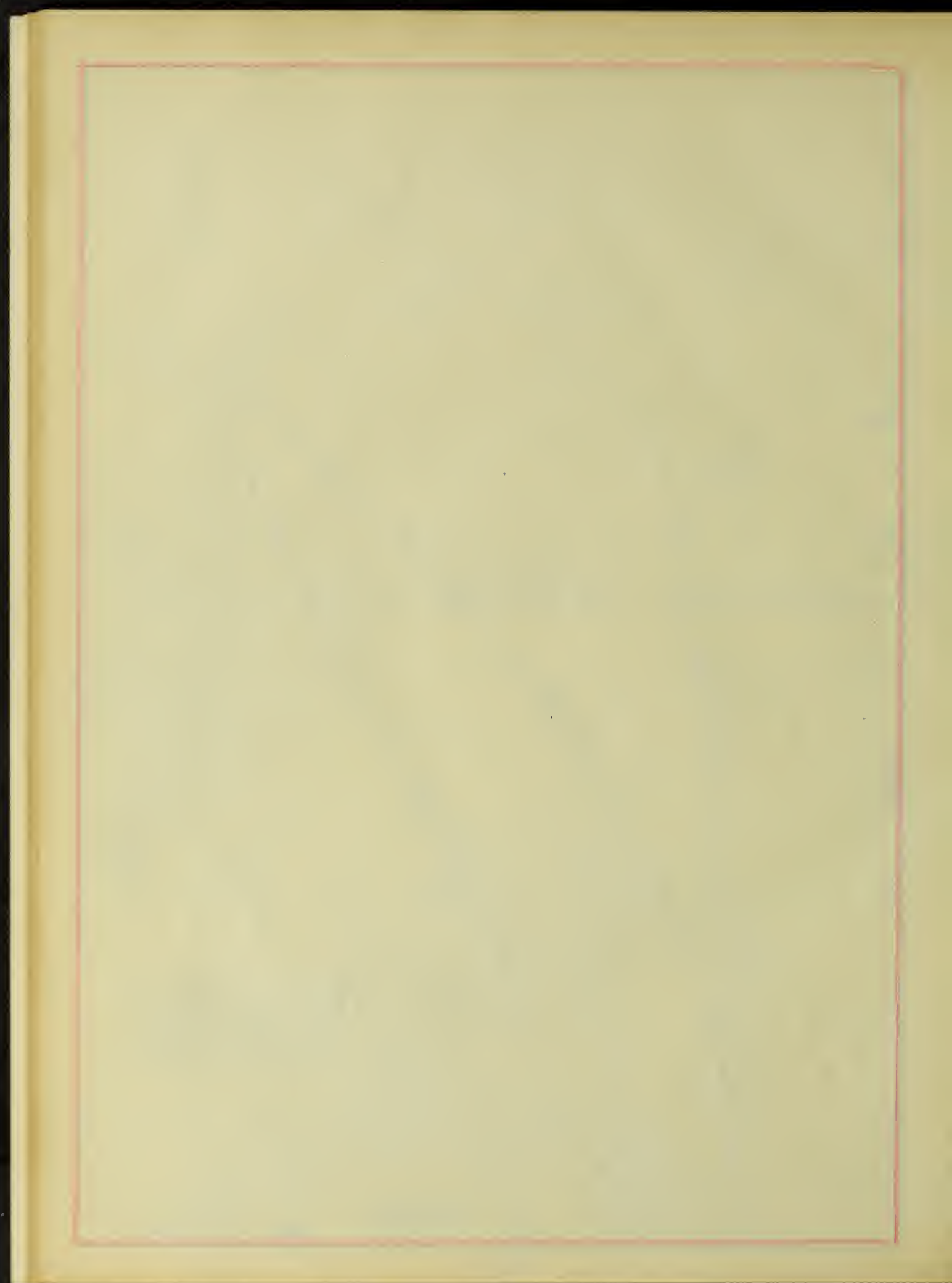
Resistance of the primary per phase as measured =  $r_0$  = .1 Ohm











# CONSTANTS.

From the above curves, the following values were taken corresponding to normal voltage.

$$E = 110 \quad I_{oo} = 12.2 \text{ amps.} \quad \text{Watts} = 380 \quad I_r^2 = 44.7$$

$$\text{Total } W_h = 195.$$

Reduced to values per phase these are :-

$$E' = 63.5 \quad I_{oo} = 12.2 \quad W' = 126.7 \quad I_r^2 = 14.9$$

$$W_h' = 65.$$

$$I_h = 65/63.5 = 1.02 \text{ amps.}$$

$$g = I_h/E' = .0161$$

$$I_m = \sqrt{12.2^2 - 1.02^2} = 12.15 \text{ amps.}$$

$$b = I_m/E' = 12.15/63.5 = .191$$

$$Y = .0161 + j.191$$

$$F = 47 \text{ watts.}$$

## RESULTS OF BLOCKED ROTOR TEST.

E volts	I amps.	Watts total	R W/3I <sup>2</sup>	Z E/1.73I	I <sub>r</sub> <sup>2</sup> 3I <sup>2</sup> .01	I <sub>r</sub> <sup>2</sup> 3I <sup>2</sup> .1122
9.95	11.8	98	.231	.485	41.7	51.
12.2	14.5	145	.23	.488	63.3	77.
13.5	16.0	174	.227	.485	76.9	93.7
15.2	18.0	215	.221	.488	97.3	118.5
16.8	19.9	260	.217	.488	118.8	145.
18.4	21.9	320	.222	.486	144.	173.
20.8	24.6	395	.219	.488	182.	222.
22.3	26.4	455	.218	.488	209.5	254.
23.9	28.2	515	.216	.488	239.5	291.
25.2	29.8	580	.217	.488	267.	325.
		Mean	.222	.488		





# BLOCKED ROTOR TEST OF THREE PHASE INDUCTION MOTOR

600 120

500 100

WATTS.  
VOLTS.

300 60

200 40

100 20

20

10

60

15

AMPERES. 100

120

AMPERES. 25

30

VOLTS.

WATTS.

IR SECONDARY.

IR PRIMARY.

THE  
J. H.  
MAYNARD



From the above data,

$$\text{mean } R = .222$$

$$\text{mean } Z = .488$$

$$r_1 = R - r_0 = .22 - .1 = .122 \text{ ohms.}$$

$$X = \sqrt{Z^2 - R^2}$$

$$= \sqrt{.488^2 - .222^2} = .437 \text{ ohms.}$$

$$x_0 = x_1 = .437/2 = .217 \text{ ohms.}$$

#### CONSTANTS.

The constants of the three phase induction motor having been determined, the characteristics were calculated by the analytical equations previously developed. The following constants are used in the calculations.

$$r_0 = .100$$

$$r_1 = .122$$

$$x_0 = .217$$

$$x_1 = .217$$

$$g = .0161$$

$$b = .191$$

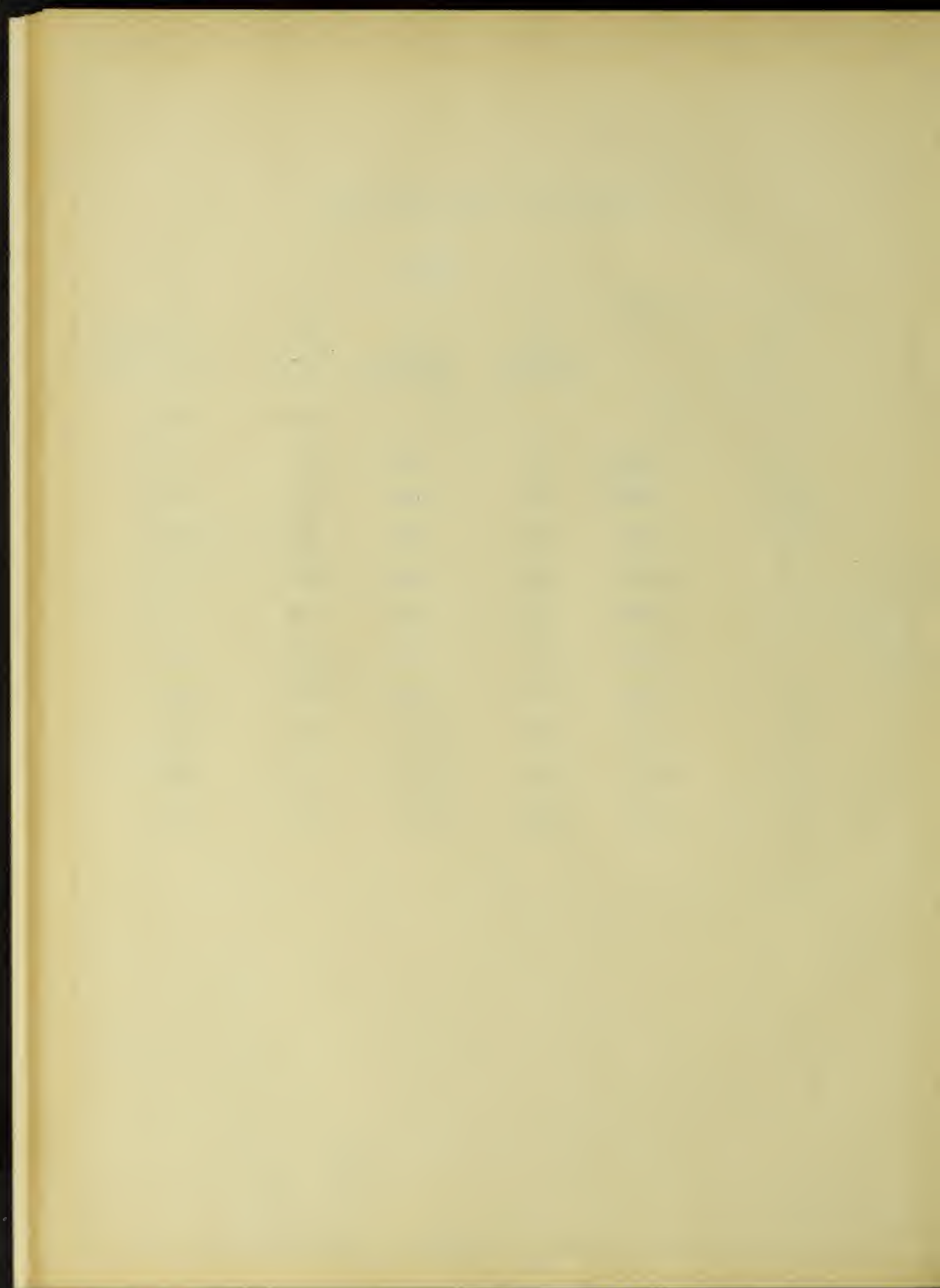
The calculations will be found on the following pages for the three phase induction motor.





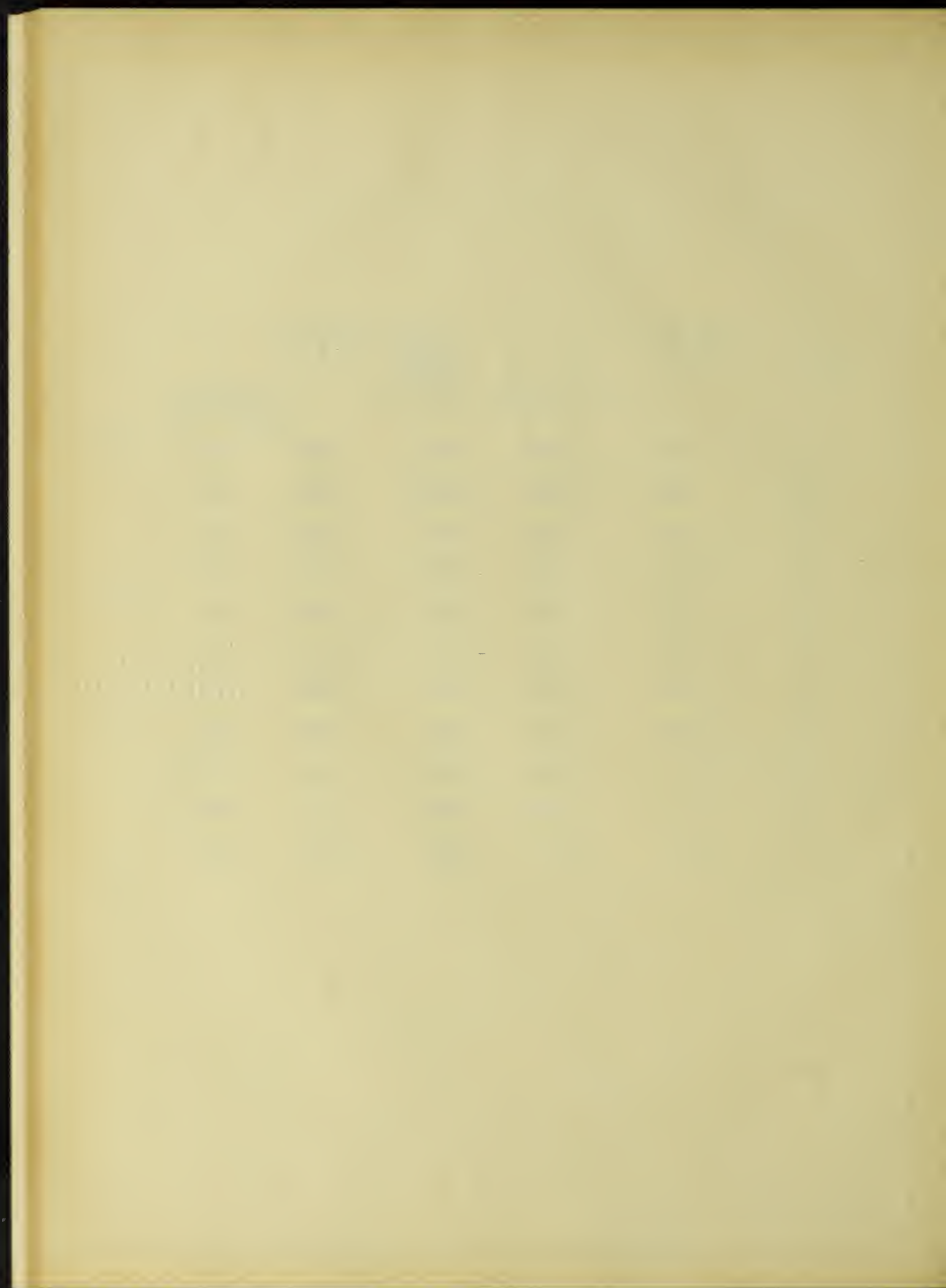
# CALCULATIONS OF THREE PHASE MOTOR.

Slip	$r_1^2 + s^2 x_1^2$	$a_1$ $\frac{sr_1^2}{r_1^2 + s^2 x_1^2}$	$a_2$ $\frac{s^2 x_1^2}{r_1^2 + s^2 x_1^2}$	$b_1$ $a_1 + g$	$b_2$ $a_2 + b$
.0	.0	.0	.0	.0161	.191
.02	.014899	.164	.00582	.18	.1968
.04	.014955	.326	.0232	.342	.2142
.06	.01505	.486	.0518	.502	.2428
.08	.01518	.644	.0916	.66	.2826
.1	.01535	.795	.1413	.811	.3323
.2	.01677	1.46	.517	1.476	.708
.4	.02243	2.175	1.545	2.191	1.736
.6	.0319	2.295	2.45	2.311	2.641
.8	.0451	2.16	3.08	2.176	3.271
1.	.0620	1.965	3.50	1.981	3.691





Slip	$\sqrt{b_1^2 + b_2^2}$	$c_1$	$c_2$	$\sqrt{c_1^2 + c_2^2}$	$e$
		$1 + b_1 r_0 + b_2 x_0$	$b_2 r_0 - b_1 x_0$		$\frac{E_0}{\sqrt{c_1^2 + c_2^2}}$
.0	.192	1.0431	.01561	1.043	60.8
.02	.2665	1.0607	- .01937	1.061	59.8
.04	.404	1.0818	- .0528	1.083	58.5
.06	.558	1.1029	- .0847	1.108	57.3
.08	.717	1.1274	- .1149	1.133	56.
.1	.875	1.1533	- .1429	1.161	54.6
.2	1.637	1.3013	- .249	1.325	47.9
.4	2.795	1.5961	- .301	1.625	39.
.6	3.51	1.8051	- .237	1.82	34.9
.8	3.93	1.9276	- .145	1.93	32.9
1.	4.19	2.00	- .061	2.01	31.6



Slip.	$I_o$	$T_s$	$T$	$P_1$	$P_o$
	$e^2(b_1^2 - b_2^2)^{1/2}$	$(1 - s)e^2a_1^2$	$.023 T_s$	$(1 - s)^2e^2a_1^2$	$(b_1c_1 - b_2c_2)e^2$
.0	11.69	0	0.0	0	44.3
.02	15.95	607	13.95	585	670.
.04	23.6	1168	26.85	1120	1230.
.06	32.	1660	38.2	1560	1750.
.08	40.1	2021	46.5	1860	2235.
.1	47.8	2370	54.5	2135	2650.
.2	78.5	3360	77.	2690	3990.
.4	109.5	3320	76.4	1990	4520.
.6	125.	2800	64.4	1120	4330.
.8	129.	2335	53.6	467	4020.
1.	132.5	1965	45.2	0	3735.



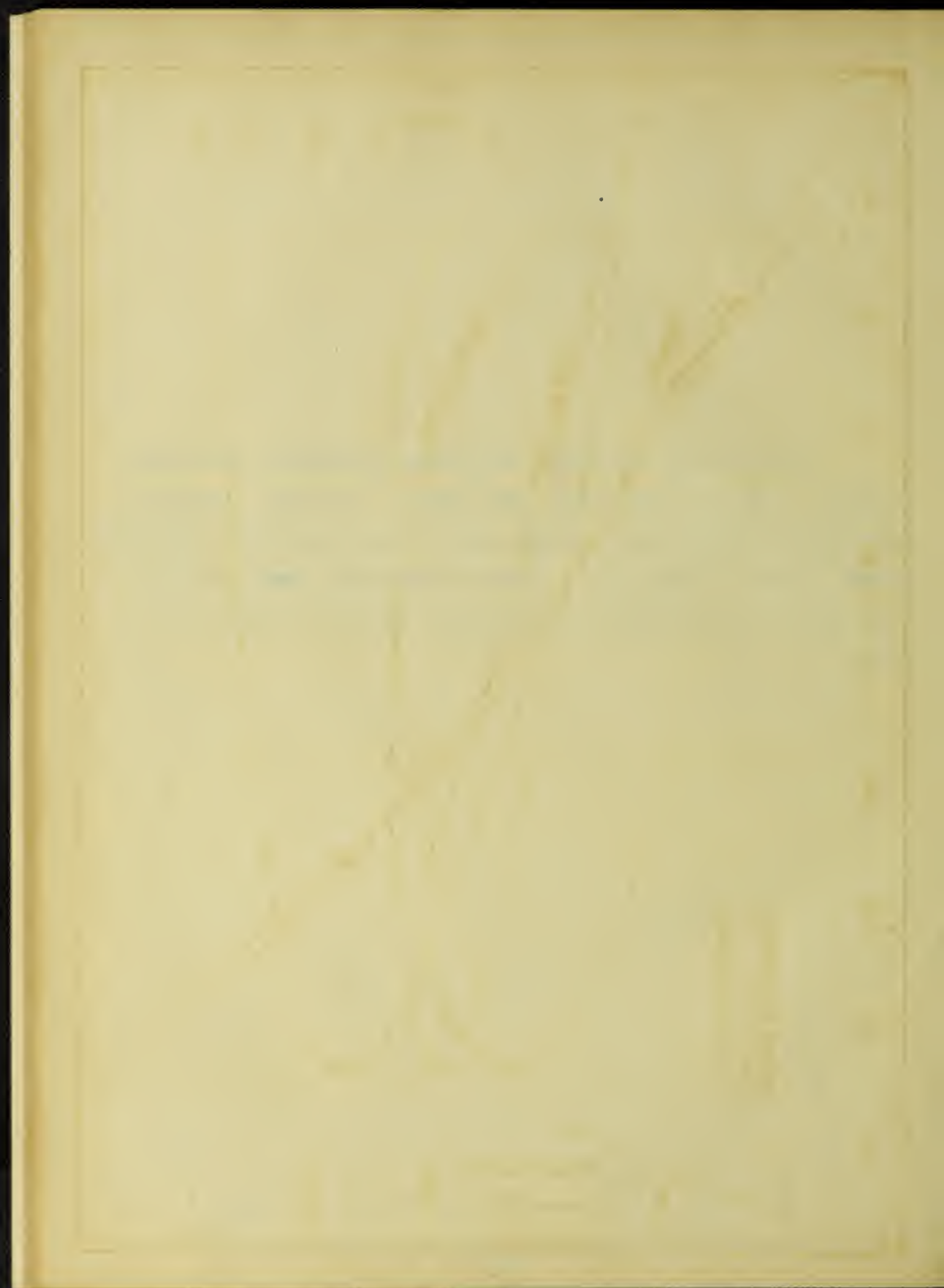


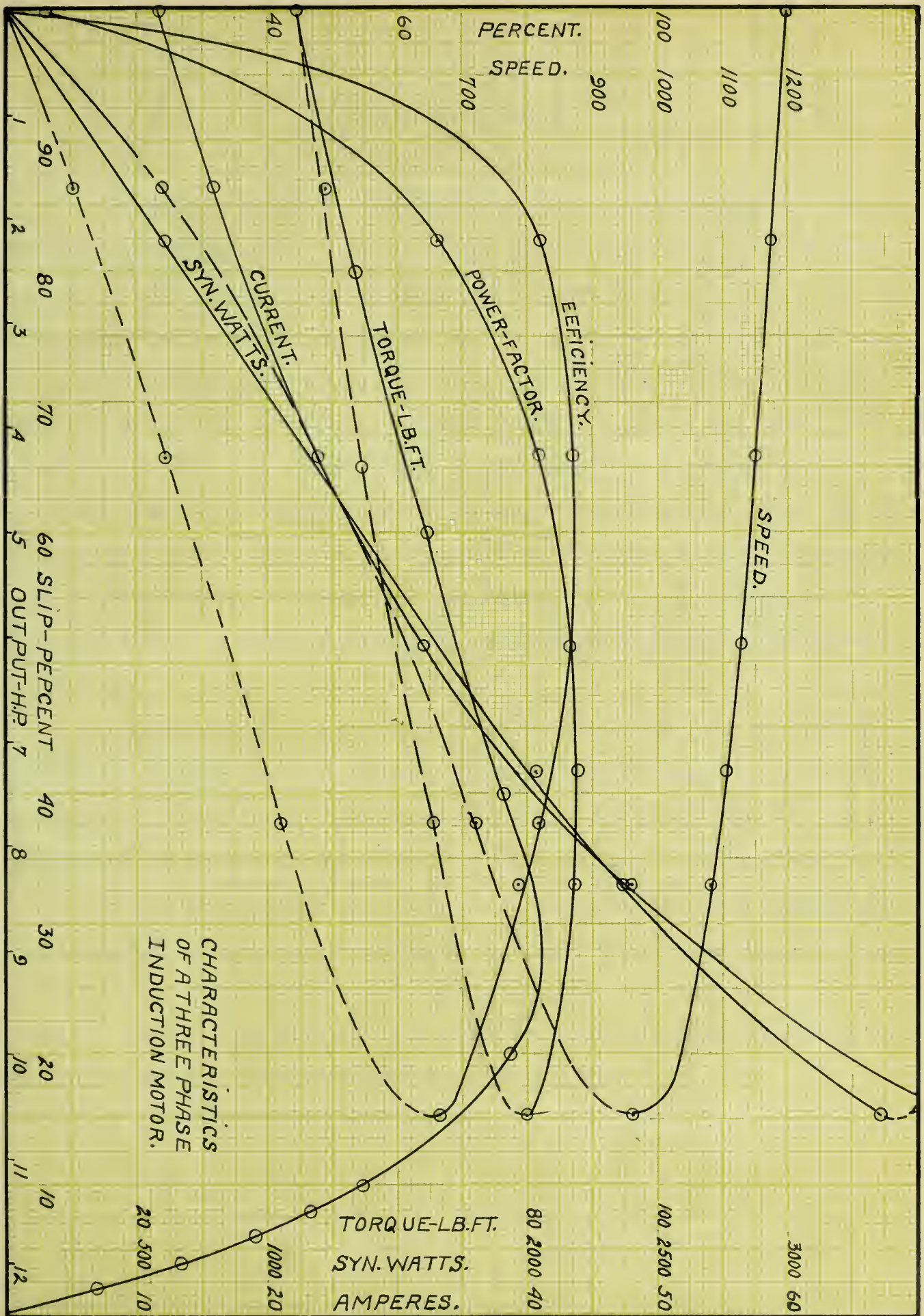
Slip	$P_a$	$P_1 - f$	Eff.	P.F.	H.P.	Speed.
	$E_o I_o$		$\frac{P_1 - f}{P_o}$	$\frac{P_o}{P_a}$	$\frac{3(P_1 - f)}{746}$	1200(1 - s)
.0	742	0	0	.06	0	1200
.02	1015	548	.82	.66	2.2	1175
.04	1500	1073	.87	.82	4.32	1150
.0	2030	1513	.865	.863	6.09	1128
.08	2540	1813	.812	.88	7.30	1103
.1	3040	2088	.788	.872	8.4	1080
.2	4990	2643	.664	.80	10.63	960
.4	6920	1943	.42	.654	7.81	720
.6	7950	1073	.248	.545	4.32	480
.8	8190	420	.104	.49	1.69	240
1.	8410	0	.0	.445	.0	0





From the above calculations the following characteristic curves were plotted :- First, true efficiency, power factor, line current, torque in synchronous watts, and speed, against output in horse power as abscissa ; second, torque in pound feet against slip in percent of synchronous speed as abscissa. These curves will be discussed in comparison with those for the single phase motor.







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### SINGLE PHASE OPERATION.

Most of the constants to be used in calculating the characteristics of a three phase induction motor, operating single phase, differ from the three phase constants. However they cannot be determined experimentally as in the case of a three phase motor on account of several features peculiar to the single phase motor.

In a single phase motor running near synchronism, the revolving magnetic field is very similar to that of a polyphase motor. It differs from that of the latter in that part of it, the so called "speed field", is furnished by the secondary, thus requiring a secondary magnetizing current. At different speeds the frequency of this current will vary below double frequency and with it the field furnished by the secondary. A copper loss would also occur in the secondary and it would be difficult to separate the losses. The running light test is therefore not feasible.

When the motor is at standstill, the flux oscillates back and forth in one direction and does not have the revolving field characteristics. The leakage flux would be different in this condition than when in operation, and thus the reactance of the operating motor would differ from that obtained in the test.

The single phase constants must be obtained theoretically by consideration of the change in conditions which accompanies the change from three phase to single phase operation. This requires an extended study of the induction motor and is beyond the scope of this problem. The following relations between three phase and single phase constants have been given by Dr. E. J. Berg.





## Single phase value

## Three phase value

$3 r_o$	$r_o$
$3 x_o$	$x_o$
$r_l$	$r_l$
$x_l$	$x_l$
$1.73 I_m$	$I_m$
$1.73 I_h$	$I_h$

## THEORY OF SINGLE PHASE MOTOR.

The speed field of the single phase motor running at synchronism is in quadrature with the primary magnetic field. This happens as follows :- the secondary conductors cut the primary flux as they rotate. The currents induced in the conductors, lag  $90^\circ$  behind the magnetism and are carried into space  $90^\circ$  by the synchronous rotation, before they reach their maximum. The field thus produced is in quadrature with the primary exciting flux. At speeds below synchronism, these currents are carried less than  $90^\circ$  and the quadrature field is less. At zero speed there is no component of secondary flux in quadrature and consequently no torque. The torque of the single phase motor is proportional to the product of the power component of secondary current and the field flux in quadrature therewith just as in the three phase motor. But the quadrature field varies as  $1 - s$  and is not proportional to  $e$  but to  $(1 - s)e$ , hence,

$$T_s = (1 - s)e \cdot ea_1 = (1 - s)e^2 a_1$$

$$P_1 = (1 - s)T_s = (1 - s)^2 e^2 a_1$$

Using the single phase constants as previously given, the single phase characteristics were calculated by means of the equations derived for the three phase, <sup>motor</sup> with the exception of  $T_s$  and  $P_1$ , the equations for which have just been given.



# CALCULATIONS OF A SINGLE PHASE MOTOR.

Slip	$r_1^2 + s^2 x_1^2$	$a_1$ $\frac{sr_1}{r_1^2 + s^2 x_1^2}$	$a_2$ $\frac{s^2 x_1^2}{r_1^2 + s^2 x_1^2}$	$b_1$ $a_1 + g$	$b_2$ $a_2 + b$
.0	.0	.0	.0	.0161	.191
.02	.01489	.164	.00582	.1800	.1968
.04	.01495	.326	.0232	.3420	.2142
.06	.015	.486	.0518	.5020	.2428
.08	.01518	.644	.0916	.6600	.2826
.1	.01535	.795	.1413	.8110	.3323
.2	.01677	1.460	.517	1.4760	.708
.4	.02243	2.175	1.545	2.1910	1.736
.6	.0319	2.295	2.45	2.3110	2.641
.8	.0451	2.16	3.08	2.1760	3.27
1.	.062	1.965	3.50	1.981	3.69





Slip	$\sqrt{b_1^2 + b_2^2}$	$c_1$	$c_2$	$\sqrt{c_1^2 + c_2^2}$	$e$
		$1 + b_1 r_0 + b_2 x_0$	$b_2 r_0 - b_1 x_0$		$\frac{E_0}{\sqrt{c_1^2 + c_2^2}}$
.0	.192	1.1293	.0467	1.13	97.3
.02	.2665	1.182	-.0582	1.185	93.2
.04	.404	1.245	-.1588	1.26	87.5
.06	.558	1.3087	-.254	1.338	82.4
.08	.717	1.382	-.345	1.44	76.
.1	.875	1.461	-.47	1.535	71.6
.2	1.637	1.9039	-.747	2.04	53.9
.4	2.795	2.788	-.91	2.94	37.4
.6	3.51	3.415	-.718	3.49	31.5
.8	3.93	3.7828	-.45	3.78	29.1
1.	4.19	3.997	-.21	3.96	27.8



Slip	$I_o$ $e^2(b_1^2 + b_2^2)^{1/2}$	$T_s$ $(1 - s)e^2a_1^2$	$T$ .023 $T_s$	$P_1$ $(1 - s)^2e^2a_1$	$P_o$ $(b_1c_1 + b_2c_2)e^2$
.0	18.68	0	0	0	258
.02	24.65	1390	32.1	1360	1740
.04	35.2	2390	55.	2295	2990
.06	45.9	3100	71.4	2910	4060
.08	55.5	3440	76.7	3160	4750
.1	63.	3690	84.8	3320	5300
.2	78.3	3390	78.	2710	6650
.4	104.5	1825	42.	1090	6350
.6	110.5	910	21.	364	5940
.8	114.2	360	8.42	73	5680
1.	115.	0	0.	0	5400

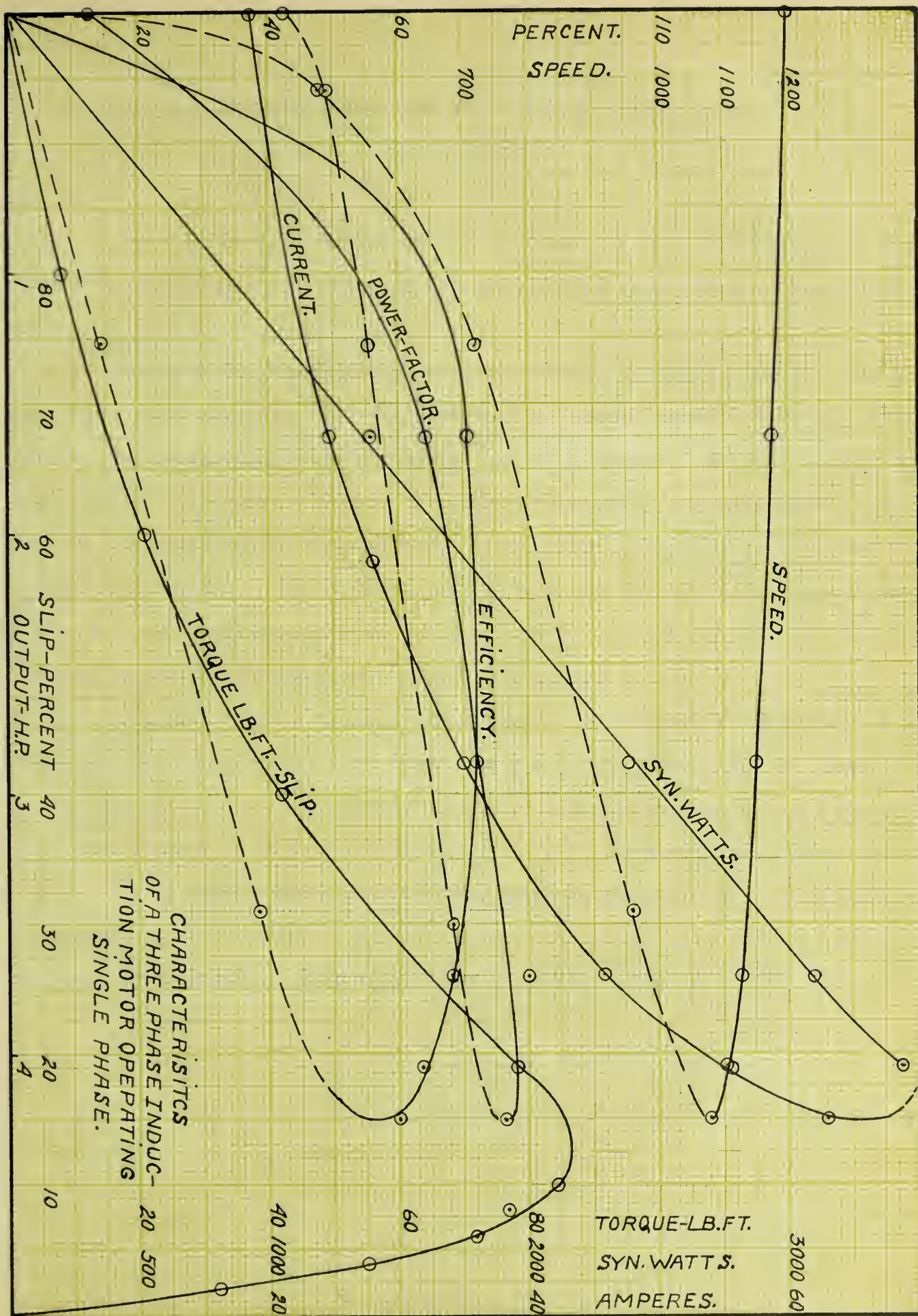




Slip.	$P_a$ $E_o I_o$	$P_1 - f$	Eff. % $\frac{P_1 - f}{P_o}$	P.F. $\frac{P_o}{P_a}$	H.P. $\frac{P_1 - f}{746}$	Speed.
.0	2050	0	0	.123	0	1200
.02	2720	1220	70.5	.64	1.63	1175
.04	3870	2155	72.	.72	2.88	1150
.06	5050	2771	68.2	.805	3.71	1128
.08	6050	3020	63.5	.785	4.05	1103
.1	6930	3180	60	.765	4.26	1080
.2	9720	2570	38.6	.685	3.45	960
.4	11500	950	14.9	.552	1.27	720
.6	12150	224	3.77	.49	.3	480
.8	12600	00	0.	.45	.0	240
1.	12700	0	0.	.425	.0	0











## DISCUSSION OF RESULTS OF THREE PHASE AND SINGLE PHASE CALCULATIONS.

A comparison of curves plotted from the calculations, will bring out the differences between the three phase and single phase characteristics of the induction motor.

The thing of principle importance is the ratio of power outputs. The maximum three phase output is 10.6 H.P. and that for single phase is 4.3 H.P. The ratio of maximum output is then  $4.3/10.6$  or 40.5 percent. This ratio is not of much practical importance since the currents at which these maximums outputs occur are 63 and 78.5 amperes respectively, or about three times <sup>the</sup> normal full load current of 28 amperes. The output at normal full load current is for three phase, 5.2 H.P. and for single phase is 2.1 H.P. the ratio being  $2.1/5.2$  or 40.5 percent, which is the same as for the maximum outputs.

The three phase efficiency curve rises to its maximum of 88 percent at the actual rated capacity of 5 H.P. and then gradually decreases to 66 percent at the maximum output. The maximum single phase efficiency is somewhat lower, being 72 percent at 2.9 H.P. and falling to 60. percent at maximum output.

It is interesting to note that the speed at maximum load is considerably greater for the single phase motor. That is, the drop in speed from no load to full load is less and the speed regulation is therefore better.



# ACTUAL TESTS.

Prony brake tests were made on the motor operating first as a three phase motor and then as a single phase motor. The voltage and frequency were maintained constant and torque, watts, amperes, and speed were measured for various loads. The single phase motor was loaded to its breakdown point but the three phase motor could not be loaded to breakdown on account of the excessive current required. From the data the H.P. output, efficiency, and power factor, were calculated as follows.

$$\text{H.P.} = \frac{2 \pi R \cdot \text{lbs.} \cdot \text{R.P.M.}}{12 \cdot 33000} = .0000794 \text{ lbs.} \cdot \text{R.P.M.}$$

where R is the radius of the pulley in inches plus one half the diameter of the rope.

$$\text{Eff.} = \frac{\text{H.P.} \cdot 746}{\text{watts intake}}$$

$$\text{P.F.} = \frac{\text{watts intake}}{1.73 \text{ E.I.}}$$

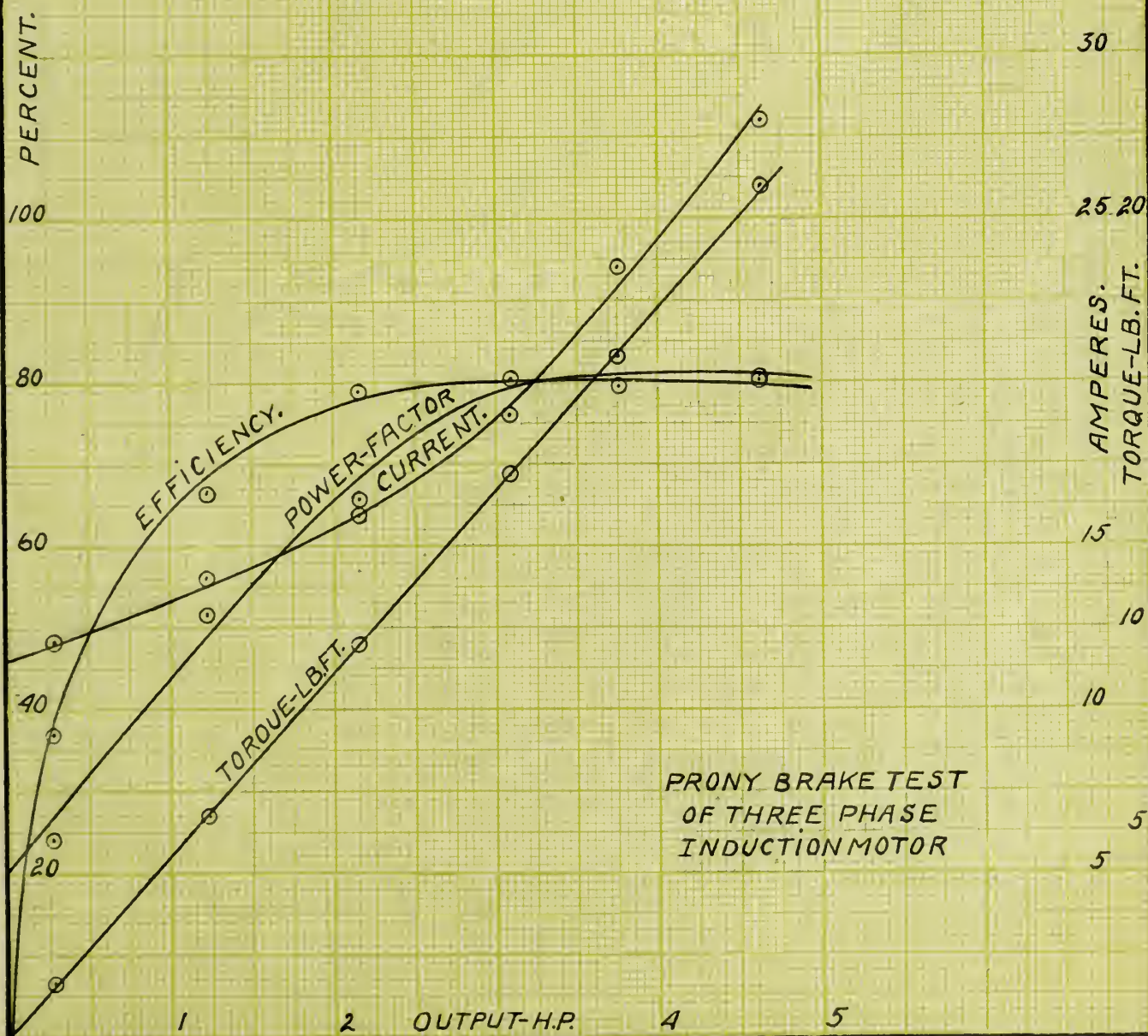
$$\text{Torque in lb.ft.} = \frac{\text{lbs.} \cdot R}{12} = .416 \text{ lbs.}$$

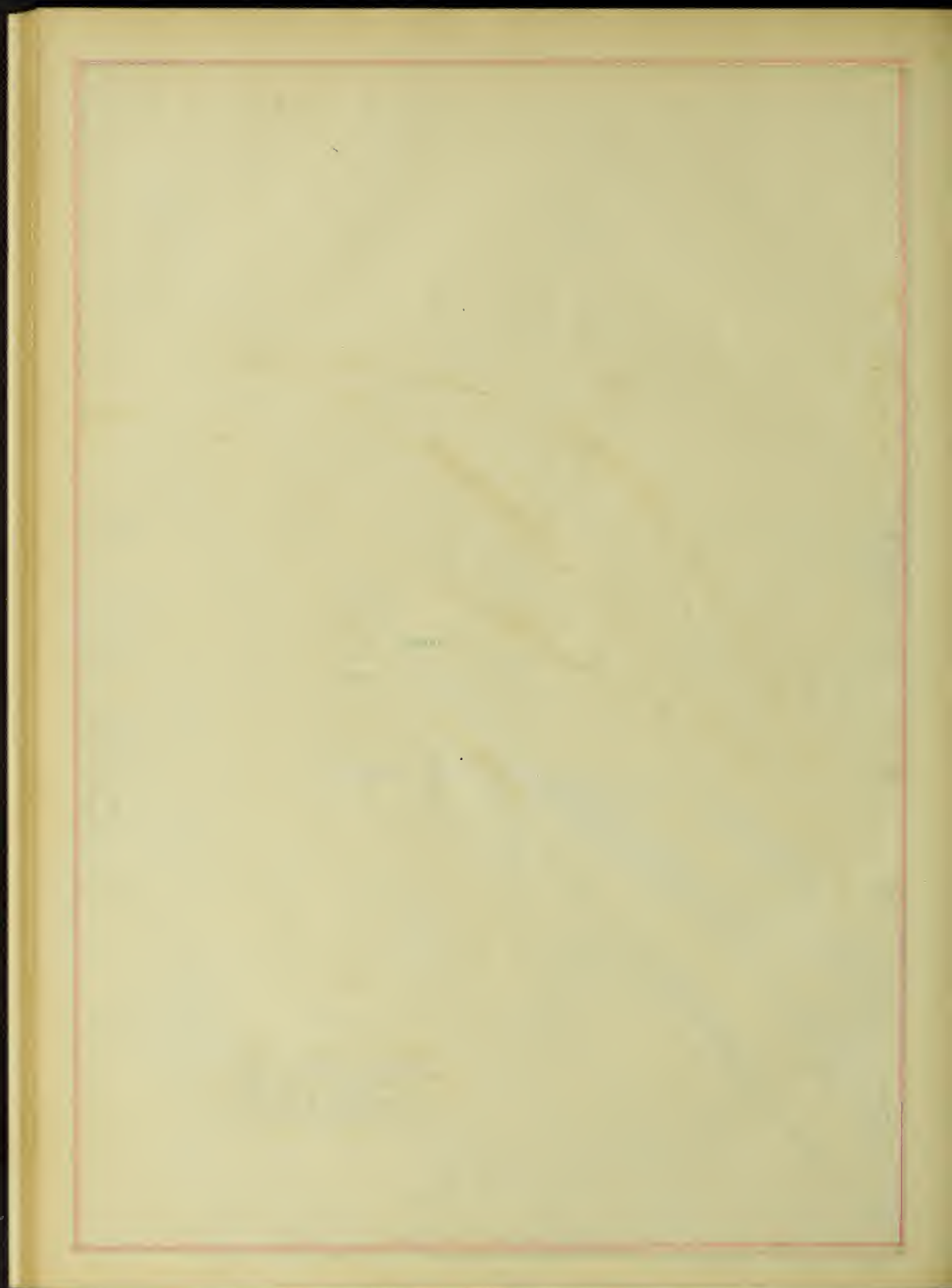
## THREE PHASE DATA AND CALCULATIONS.

E = 110 Volts									
T	I	W	T	R.P.M.	H.P.	Out put. watts	Eff.	Volt -amp.	P.F.
lbs.	amps.	watts	lb.ft.						
3.	12.4	570	1.25	1190	.28	212	.37	2360	.241
13.	14	1370	5.4	1185	1.22	910	.664	2670	.514
23	16.	2030	9.6	1180	2.16	1610	.79	3050	.655
33	19.	2900	13.7	1175	3.08	2300	.79	3620	.802
40	23.5	2550	16.6	1170	3.72	2780	.78	4480	.793
50	28.	4300	20.8	1165	4.63	3450	.8	5340	.606

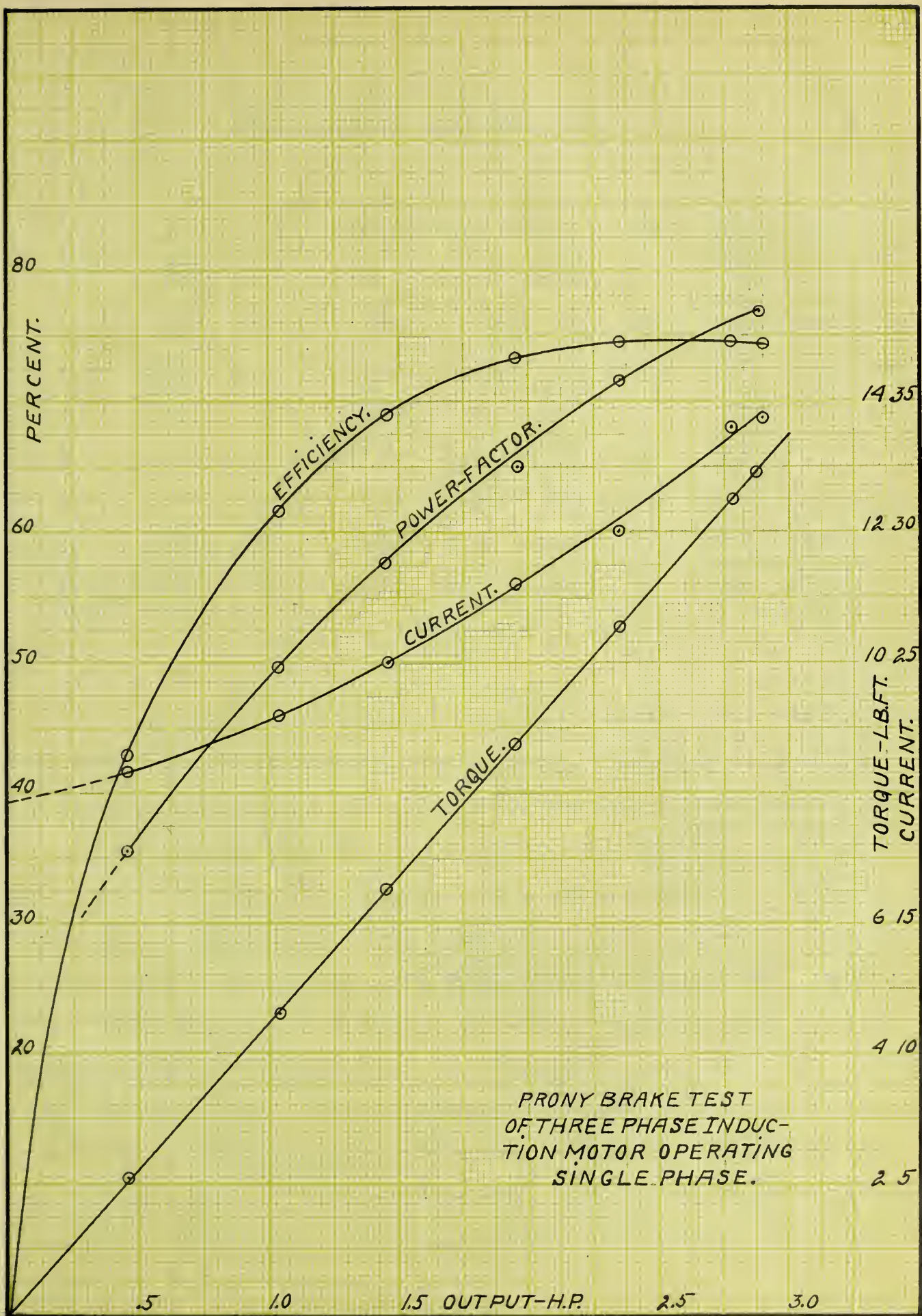












1000  
(1000)



# SINGLE PHASE DATA AND CALCULATIONS.

I amps.	W input watts	T lb.	R.P.M.	H.P.	Out put watts.	Eff.	Volt amp.	P.F.	T lb.ft.
21	820	5	1190	.472	352	.43	2310	.355	2.08
23	1260	11.	1184	1.04	775	.615	2530	.498	4.58
26	1580	15.6	1179	1.46	1090	.69	2750	.575	6.5
28	2000	21.	1174	1.96	1460	.73	3080	.65	8.75
30	2360	25.3	1171	2.36	1760	.745	3300	.715	10.55
34	2780	30.	1167	2.78	2075	.745	3740	.745	12.5
34.3	2900	31.	1167	2.88	2150	.74	3780	.767	12.9

E = 110 Volts.

From this data curves were plotted with efficiency, power factor, current intake, and torque against horse power output. Comparing the three phase and single phase curves at normal current it is seen that the three phase output is 4.63 H.P. and the single phase is 1.96 H.P., the ratio being 1.96/4.63 or .423. Since the three phase motor could not be run to breakdown, the maximum output could not be obtained. Comparisons will therefore be made from normal current values. The efficiencies are 80 percent and 73 percent; P.F. .806 and .65; torque 20.8 and 8.75 lb.ft. The values of exciting current are found from the current curve at zero load. The three phase value is 11.5 amps. and the single phase is 20 amps. It is interesting to note that the ratio is 20/11.5 or 1.74, that is, the single phase exciting current is nearly 1.73 times the per-phase exciting current of the three phase motor, which was the theroretically determined ratio. The ratio of single phase breakdown load to normal three phase



load is 2.88/4.6 or .625 . Having thus made comparisons between the three phase and single phase characteristics of the induction motor, from both theoretical and actual considerations, it is interesting to compare the theoretical results with the actual. First considering the three phase motor, the results are as follows;

### THREE PHASE.

Normal current.

	Calculated	Actual
H.P. - - - - -	5.2 - - - - -	4.6
Eff. - - - - -	.88 - - - - -	.8
P. F. - - - - -	.85- - - - -	.8
I <sub>oo</sub> - - - - -	11.7 - - - - -	<del>11.5</del>

### SINGLE PHASE.

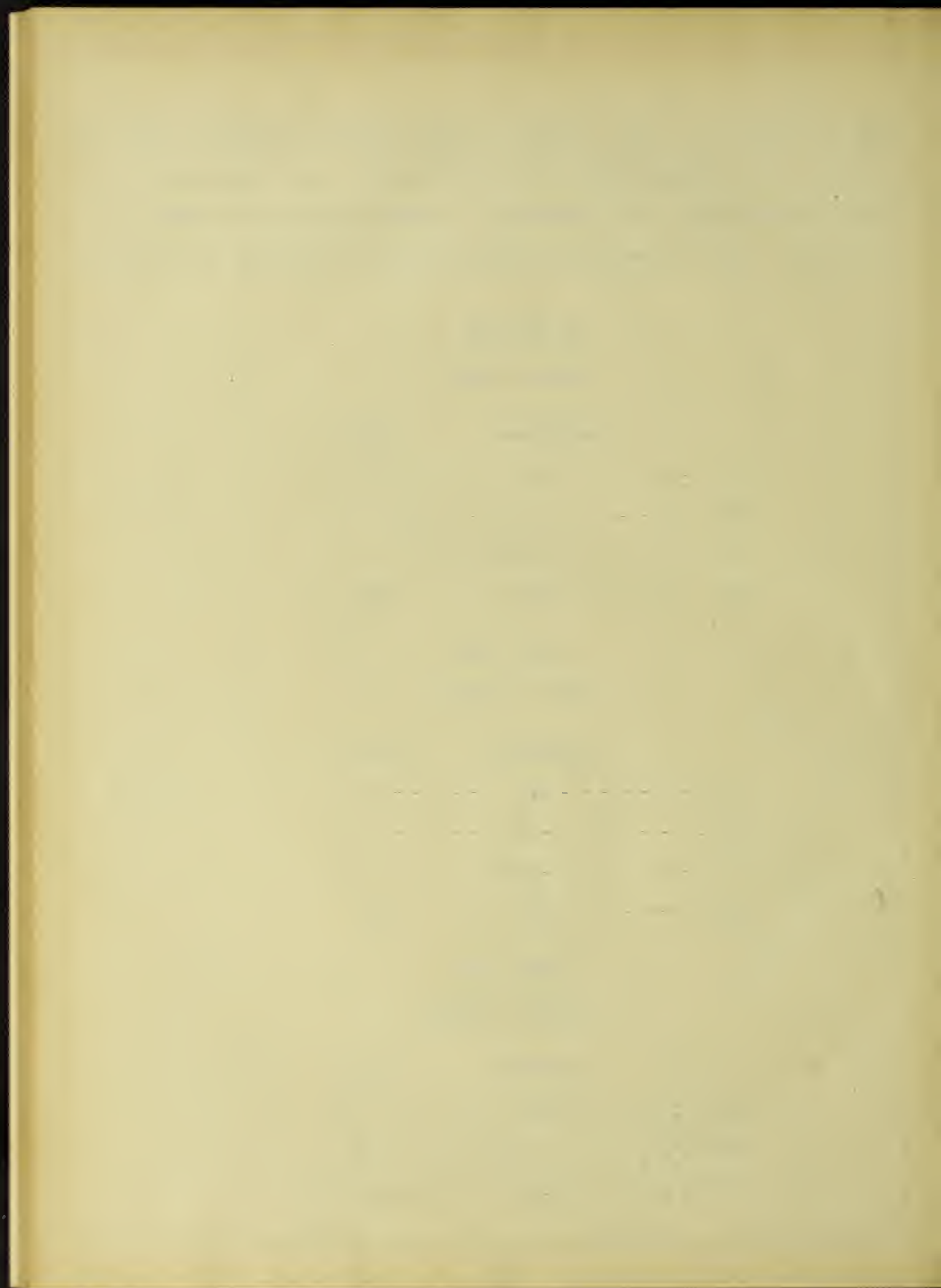
Normal current.

	Calculated	Actual.
H.P. - - - - -	2.1 - - - - -	1.96
Eff. - - - - -	.71 - - - - -	.73
P.F. - - - - -	.68 - - - - -	.65
I <sub>oo</sub> - - - - -	18.7 - - - - -	20.

### THREE PHASE.

Maximum values.

	Calculated	Actual
H.P. - - - - -	10.6 - - - - -	-----
Eff. - - - - -	.87 - - - - -	.8
P.F. - - - - -	.88 - - - - -	.8





# SINGLE PHASE.

Maximum values.

	Calculated	Actual
H.P. - - - - -	4.3 - - - - -	2.9
Eff. - - - - -	.82 - - - - -	.745
P.F. - - - - -	.805 - - - - -	.765

From these tables it is seen that the actual values are somewhat lower than the theoretical but that the percentage variation is practically constant in all cases. Various conditions occur during operation to cause the losses to be greater than those assumed and hence to make the actual characteristics inferior to those obtained theroretically. Considering the fact that the prony brake is not accurate within five percent, the results compare reasonable will.

## CONCLUSIONS.

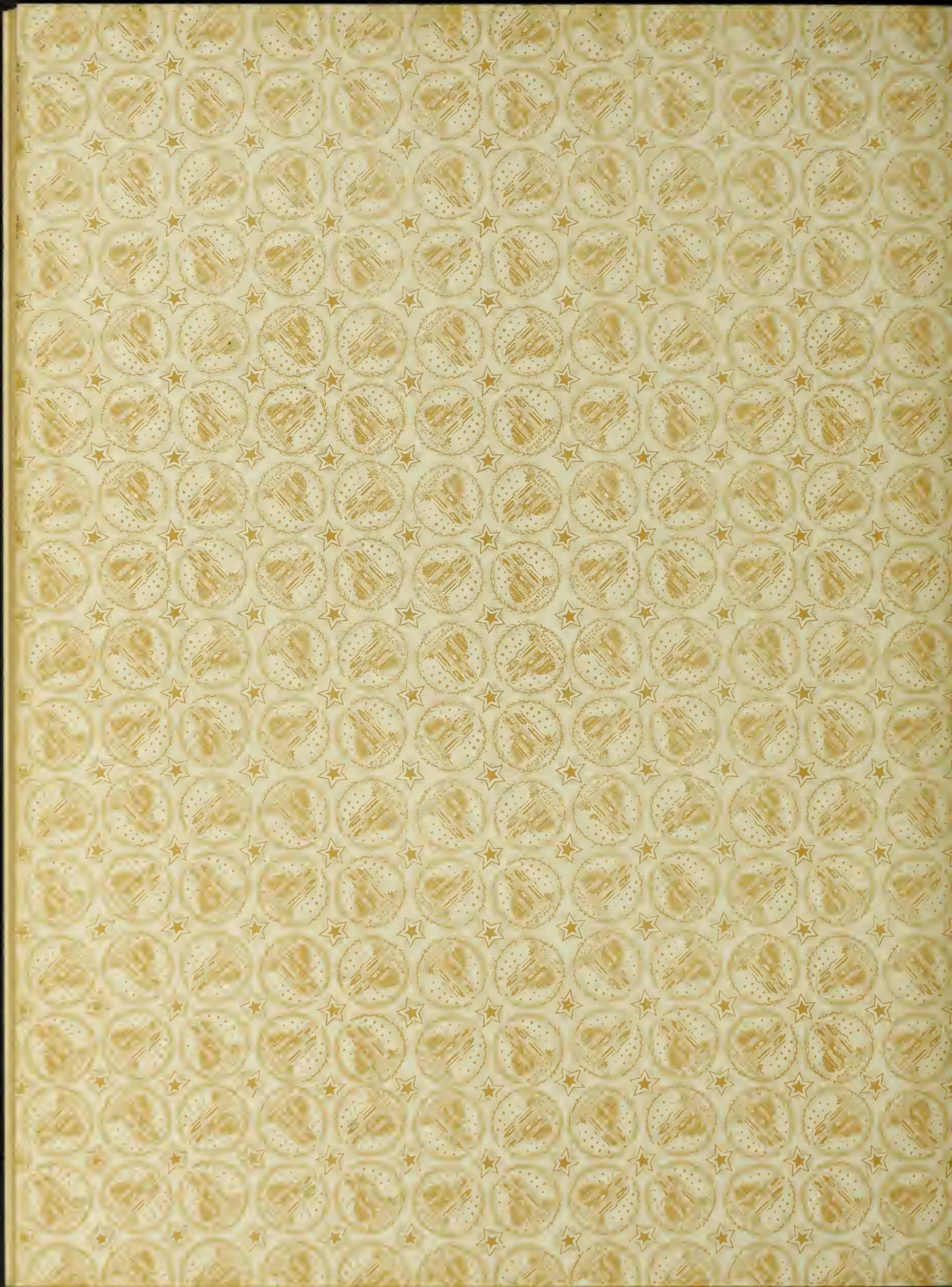
It has been shown in the foregoing considerations that a three phase motor operating as a single phase motor from two wires of a three phase power system, with the same phase current, will give approximately 40 percent of the power it would give when running as a three phase motor. Since the internal losses are considerably less due to the absence of copper loss in one of the coils, the single phase motor will carry a heavier current without exceeding its heating capacity and will thus furnish more power. Under this condition it would likely carry 50 percent of the three phase normal current load. It is found from the above that the actual breakdown load was about 62 percent of normal phase three phase load. Suppose the motor to be running from three phase power and carrying a constant load, which would likely be near the rating of the machine, say for instance



75 percent. Then a load of 75 percent of 4.6 or 3.5 H.P. would be carried by the motor. If under these conditions, a line became disconnected or a phase opened in any manner, the load would exceed the single phase breakdown point and the motor would stop, thus drawing an enormous current and blowing the fuses. Suppose again the motor is operating three phase with a fluctuating load and that at the time of open circuit of a phase, the load is between 50 and 60 percent of normal load. The motor would continue operation as a single phase motor but under overload conditions and if continued for a very long time would burn out the motor. However with a fluctuating load any momentary rise above 62 percent would throw the motor out of step and blow the fuse, thus preventing its burning out. Thus the probability that a three phase motor would burn out is not very great.

In general it may be said that a three phase induction motor will operate successfully as a single phase motor carrying half the load with better speed regulation but with poorer P.F. and efficiency.











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